On the use of information measures and performance bounds in the ISM

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When an astronomer tries to infer a physical quantity from observations, he first has to choose among the many possible techniques: from the well-known weighted least squares and maximum likelihood estimators, to the more recent random forest and neural network estimators, through the Bayesian estimators. However, the best-performing estimator depends on the considered problem. A careful astronomer would thus need to implement all the different techniques to choose the best one for his problem. This is of course out of question because of the required time.

Information measures and performance bounds provide an alternative view of the data, independent on the choice of the estimation technique. On the one hand, for a given joint probability density function (pdf) of the observable and the physical quantity of interest, information theory allows one to quantify the dependence between the observable and the parameter of interest. On the other hand, the performance bounds provide estimation precisions for a given physical regime.

This course provides an introduction to the main information theory concepts, namely the entropy and mutual information. I will show how these information measures can be used for the selection of the most informative lines in astrophysics [1]. Then, after showing a connection between information theory and estimation theory, I will discuss several ways to define an optimal estimator. Finally, I will present the Cramér-Rao lower bound which can be used 1) to quantify the gain in precision provided by new observations, and 2) to find out whether all the information from the data has been extracted, and 3) to build an interval of confidence [2,3].

During the hands-on sessions, you will use these concepts on actual examples. Some Python codes will be provided to generate simulated data.

Bibliography

Application to ISM studies:

[1] Einig, L., Palud, P., Roueff, A., et al. 2024, A&A, 691.

[2] Roueff, A., Pety, J., Gerin, M., et al. 2024, A&A, 686.

[3] Segal, L., Roueff, A., Pety, J., et al. 2024, A&A, 692.

Useful Textbooks:

[5] "Fundamentals of Statistical Signal Processing: Estimation Theory" by S. Kay, published by Prentice Hall.

[6] "Elements of Information Theory" by Cover & Thomas, published by Wiley.